Maths, Cognition and Logic Challenge
Mathelots
May 7, 2025
Part III

Instructions

- Please read these instructions carefully before attempting the Challenge.
- The Challenge comprises 15 free-response style problems to be completed in 75 minutes. Each problem carries 6 marks, adding to a total of 90 marks. The remaining 10 marks are for filling out the questionnaire.
- Each question has an integer answer between 0 and 999. All answers must be written as 3-digit numbers between 000 to 999 in the correct place in the Answer Booklet. If your answer to any question is a two-digit or one-digit number, such as 89 or 7, you **must** write it as 089 and 007 respectively on the Answer Booklet. No overwriting will be allowed!
- The Challenge is accompanied by a questionnaire to help us take your feedback, collect valuable research data, and calibrate our future contests. Your sincere submissions will go a long way! All sincere answers are valid and you get one mark for each answered question on this questionnaire.
- Please write your full name, your or your guardian's e-mail address and phone number, your school name, your grade and the Part you are taking (I, II or III to be written as 001, 002, 003, respectively) on the Answer Booklet. The winners and online participants will be notified accordingly.
- The top 30 contestants from each Part will be invited to a three-day live workshop tentatively from May 15, 2025 to May 17, 2025.
- All participants will also be able to join a live-stream of the sessions at our workshop. For more details, please go to our website www.mathelots.com as we make updates in the coming week. You may reach out to us directly at info@mathelots.com at any time!
- *The problems are designed to be challenging and any progress you make is to be celebrated. We really hope you enjoy the Challenge!*

Question Paper

Problem 1. Find the value of $\sqrt{111556}$.

Problem 2. Find the value of

$$\frac{10 \cdot 20 \cdot 30 + 20 \cdot 40 \cdot 60 + 40 \cdot 80 \cdot 120 + 70 \cdot 140 \cdot 210}{1 \cdot 3 \cdot 5 + 2 \cdot 6 \cdot 10 + 4 \cdot 12 \cdot 20 + 7 \cdot 21 \cdot 35}.$$

Problem 3. Hacker Bob is trying to find the smallest positive integer k with

$$2^k > 10^{15}.$$

Can you help him find this value of k?

Problem 4. Determine the last three digits of the number

$$9 + 99 + 999 + \cdots + \underbrace{999\ldots9}_{\text{999 digits total}}.$$

As an example, the last three digits of 1729 are 729, the last three digits of 23 are 023, and the last three digits of 7 are 007.

Problem 5. A plumber has a collection of sinks $s_2, \ldots, s_{100}$ that can each individually drain a full tank in $2, 3, 4, \ldots, 100$ hours, respectively. What is the smallest number of sinks he needs to drain a full tank in less than 40 minutes?

Problem 6. Find the last three digits of the sum of all distinct prime factors of the number 160401.

For example, $387 = 3 \times 3 \times 43$, so the sum of all distinct prime factors of 387 equals $3 + 43 = 46$.

Problem 7. On an island live two clans of people — the knights and the knaves. The knights always tell the truth and the knaves always lie. None of them will ever make a logically impossible statement.

One day, everyone told everyone else: "You are all knaves."

How many knights are there on the island?

Problem 8. A language school with 1000 students offers courses in English, Sanskrit and Tamil and each student must choose at least one language. A student is called *monolingual* if they choose exactly one language to study, *bilingual* if they choose exactly two languages to study, and *trilingual* if they choose all three languages to study.

It is known that there are 21 more bilingual students in the school than trilingual students. Furthermore, for each course registration, a student had to pay 100 rupees, so a monolingual student paid 100 rupees, a bilingual student paid 200 rupees and a trilingual student paid 300 rupees. If the total amount collected by the school is equal to $2,20,000$ rupees, find the number of monolingual students.

Problem 9. Find the number of integer triples (a, b, c) with $0 \leqslant a, b, c \leqslant 9$ such that $a + b + c$ is a multiple of 3.

Note. Two triples $(a, b, c), (a', b', c')$ are the same iff $a = a', b = b', c = c'$.

Problem 10. In a class, there are 19 desks, each capable of seating at most two students. Each student has seating space available. Students must choose a sport and are only allowed to choose one of cricket or football but not both. Every student who plays cricket is friends with exactly three students who play football and every student who plays football is friends with exactly two students who play cricket. It is known that 31 of the students attend mathematics class. How many students are there?

Problem 11. Alice, Bob, Carol decide to play a series of friendly table-tennis matches. They have only two table-tennis bats, so they decide that after every match, the winner of that match gets to play the third person who did not participate in the match. Each match ends decisively. At the end of the matches, it turned out that Alice played 17 matches, Bob played 15 matches and Carol played 10 matches. How many matches did Carol win?

Problem 12. Agents $\#1, \#2, \ldots, \#1001$ are assigned on an internal survey mission such that each agent monitors the activities of one other agent among them and every agent is monitored by exactly one other agent. It is known that Agent #1 monitors the Agent who is monitoring Agent #2, Agent #2 is monitoring the Agent monitoring Agent #3, and so on, at last with Agent #1001 monitoring the Agent monitoring Agent #1. Which Agent is Agent #37 monitoring?

Problem 13. Two spies communicate through a code. Each digit is assigned a unique letter from $A, B, C, D, E, F, G, H, I, J$ for each of the digits $1, 2, 3, 4, 5, 6, 7, 8, 9, 0$, in some *unknown* order. One of the spies wrote

$$\overline{ABC} + \overline{CBA} = 928$$

which is known to be correct. Please help the other spy find the maximum possible value of

$$D^2 + E^2 + F^2 + G^2 + H^2 + I^2 + J^2.$$

Problem 14. Suppose the numbers $1, 2, 3, \ldots, n$ are written on a blackboard in this order, with $n > 1000$. The numbers are written using one of two colours — either red or blue with each number coloured only once and no number left uncoloured. Let r be the largest number on a red card and b be the smallest number on a blue card. Suppose $1 < r < 1000$, r equals the sum of the labels of all the other red cards, $r + b = n$, and r equals the total number of blue cards. Find the value of r.

Problem 15. Sisyphus has written the number x on the blackboard. He may pay one rupee to perform one of two admissible operations:

- If $x > 0$, he may erase x and write the number $\frac{1}{x}$;
- If $x > 1$, he may erase x and write the number $x - 1$.

Starting with $x = \frac{10!}{11}$, let N be the smallest number of rupees he must pay to be able to write the number 0. Find the last three digits of N.

Here, $10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$.

Questionnaire

1. On a scale of 1 to 5, how would you rate this challenge in terms of *enjoyment*? (with 1 being least enjoyable contest you've ever taken to 5 being the most enjoyable contest you've ever taken)
2. On a scale of 1 to 5, how would you rate the difficulty of this challenge relative to other competitions you've taken? (with 1 being the easiest contest you've ever taken and 5 being the most difficult one you've ever taken)
3. Which were your top three favourite problems? Feel free to just write down the problem numbers in the response column.
4. Which problems did you not solve but really enjoyed thinking about? Feel free to just write down the problem numbers in the response column.
5. Among problems you did not solve, were there ones you felt you could have solved under better conditions, such as more test-time, or more preparation before the challenge? Which ones?
6. Are there any problems on the challenge that make you excited about maths? Please write down their numbers.
7. Are there questions you would want us to ask in this questionnaire that we did not? (Yes or No) Please mail it to us at info@mathelots.com
8. Did you run short of time? (Yes or No)
9. Which problems did you spend the most time on and which did you spend the least time (or none) on? Please mention the problem numbers.
10. Would you like to do a similar Challenge again in the near future?