Maths, Cognition and Logic Challenge
Mathelots
May 7, 2025
Part II

Instructions

- Please read these instructions carefully before attempting the Challenge.
- The Challenge comprises 15 free-response style problems to be completed in 75 minutes. Each problem carries 6 marks, adding to a total of 90 marks. The remaining 10 marks are for filling out the questionnaire.
- Each question has an integer answer between 0 and 999. All answers must be written as 3-digit numnbers between 000 to 999 in the correct place in the Answer Booklet. If your answer to any question is a two-digit or one-digit number, such as 89 or 7, you **must** write it as 089 and 007 respectively on the Answer Booklet. No overwriting will be allowed!
- The Challenge is accompanied by a questionnaire to help us take your feedback, collect valuable research data, and calibrate our future contests. Your sincere submissions will go a long way! All sincere answers are valid and you get one mark for each answered question on this questionnaire.
- Please write your full name, your or your guardian's e-mail address and phone number, your school name, your grade and the Part you are taking (I, II or III to be written as 001, 002, 003, respectively) on the Answer Booklet. The winners and online participants will be notified accordingly.
- The top 30 contestants from each Part will be invited to a three-day live workshop tentatively from May 15, 2025 to May 17, 2025.
- All participants will also be able to join a live-stream of the sessions at our workshop. For more details, please go to our website www.mathelots.com as we make updates in the coming week. You may reach out to us directly at info@mathelots.com at any time!
- *The problems are designed to be challenging and any progress you make is to be celebrated. We really hope you enjoy the Challenge!*

Question Paper

Problem 1. Find the value of $\sqrt{111556}$.

Problem 2. Find the value of

$$\frac{10 \cdot 20 \cdot 30 + 20 \cdot 40 \cdot 60 + 40 \cdot 80 \cdot 120 + 70 \cdot 140 \cdot 210}{1 \cdot 3 \cdot 5 + 2 \cdot 6 \cdot 10 + 4 \cdot 12 \cdot 20 + 7 \cdot 21 \cdot 35}.$$

Problem 3. Ravi commutes 10 kilometres daily to his office in 100 minutes. During his journey he spends an hour walking, twenty minutes in the train and another twenty minutes in a cab. The speed of the train is twice his walking speed. If he were to walk for an hour he would cover 20 fewer kilometres than if he took a cab for the whole hour. How long would it take Ravi to reach his office if he were to only walk? Express your answer in minutes.

Problem 4. How many times does the digit 5 occur when writing all natural numbers from 1 to $1,000$?

Problem 5. Find the smallest integer k such that $2^k > 10^{15}$.

Problem 6. Find the sum of all distinct prime factors of 159999.

For example, $387 = 3 \times 3 \times 43$, so the sum of all distinct prime factors of 387 equals $3 + 43 = 46$.

Problem 7. Suppose $\diamond$ denotes the operation

$$a \diamond b = ab + a + b$$

for any two natural numbers a and b. For example, $5 \diamond 7 = 5 \times 7 + 5 + 7 = 47$. Find the value of

$$((((1 \diamond 2) \diamond 3) \diamond 4) \diamond 5).$$

Problem 8. Determine the last three digits of the number

$$9 + 99 + 999 + \cdots + \underbrace{999 \ldots 9}_{\text{nine digits total}}.$$

As an example, the last three digits of 1729 are 729, the last three digits of 23 are 023, and the last three digits of 7 are 007.

Problem 9. A plumber has a collection of sinks $s_2, \ldots, s_{100}$ that can each individually drain a full tank in $2, 3, 4, \ldots, 100$ hours, respectively. What is the smallest number of sinks he needs to drain a full tank in less than 40 minutes?

Problem 10. On an island live two clans of people — the knights and the knaves. The knights always tell the truth and the knaves always lie. None of them will ever make a logically impossible statement.

One day, everyone told everyone else: "You are all knaves."

How many knights are there on the island?

Problem 11. A language school with 1000 students offers courses in English, Sanskrit and Tamil and each student must choose at least one language. A student is called *monolingual* if they choose exactly one language to study, *bilingual* if they choose exactly two languages to study, and *trilingual* if they choose all three languages to study.

It is known that there are 21 more bilingual students at the school than trilingual students. Further, for each course registration, a student had to pay 100 rupees, so a monolingual student paid 100 rupees, a bilingual student paid 200 rupees and a trilingual student paid 300 rupees. If the total amount collected by the school equals $2,20,000$ rupees, find the number of monolingual students.

Problem 12. At the conference of the knights and knaves clans, 1001 people gathered together. The knights always tell the truth and the knaves always lie. At the gathering, the people were numbered $A_1, A_2, \ldots, A_{1001}$ and the following statements were made:

- A_1: "A_2 is a knave."
- A_2: "A_3 is a knave."
- A_i: "A_{i+1} is a knave." for every $1 \leqslant i \leqslant 1000$.
- And finally, A_{1001}: "There are strictly more knaves than knights among $A_1, A_2, \ldots, A_{1000}$."

How many knaves were there at the gathering?

Problem 13. Alice, Bob, Carol decide to play a series of friendly table-tennis matches. They have only two table-tennis bats, so they decide that after every match, the winner of that match gets to play the third person who did not participate in the match. Each match ends decisively. At the end of the matches, it turned out that Alice played 17 matches, Bob played 15 matches and Carol played 10 matches. How many matches did Carol win?

Problem 14. Agents $\#1, \#2, \ldots, \#1001$ are assigned on an internal survey mission such that each agent monitors the activities of one other agent among them and every agent is monitored by exactly one other agent. It is known that Agent #1 monitors the Agent who is monitoring Agent #2, Agent #2 is monitoring the Agent monitoring Agent #3, and so on, at last with Agent #1001 monitoring the Agent monitoring Agent #1. Who is Agent #37 monitoring?

Problem 15. Two spies communicate through a code. Each digit is assigned a unique letter from $A, B, C, D, E, F, G, H, I, J$ for each of the digits $1, 2, 3, 4, 5, 6, 7, 8, 9, 0$, in some *unknown* order. One of the spies wrote

$$\overline{ABC} + \overline{CBA} = 928$$

which is known to be correct. Please help the other spy find the value of the product

$$(A + B + C) \times (D + E + F + G + H + I + J).$$

Questionnaire

1. On a scale of 1 to 5, how would you rate this challenge in terms of *enjoyment*? (with 1 being least enjoyable contest you've ever taken to 5 being the most enjoyable contest you've ever taken)
2. On a scale of 1 to 5, how would you rate the difficulty of this challenge relative to other competitions you've taken? (with 1 being the easiest contest you've ever taken and 5 being the most difficult one you've ever taken)
3. Which were your top three favourite problems? Feel free to just write down the problem numbers in the response column.
4. Which problems did you not solve but really enjoyed thinking about? Feel free to just write down the problem numbers in the response column.
5. Among problems you did not solve, were there ones you felt you could have solved under better conditions, such as more test-time, or more preparation before the challenge? Which ones?
6. Are there any problems on the challenge that make you excited about maths? Please write down their numbers.
7. Are there questions you would want us to ask in this questionnaire that we did not? (Yes or No) Please mail it to us at info@mathelots.com
8. Did you run short of time? (Yes or No)
9. Which problems did you spend the most time on and which did you spend the least time (or none) on? Please mention the problem numbers.
10. Would you like to do a similar Challenge again in the near future?